Bonn Problem-Solving Seminar, 2013

Prerequisites

András Frank
frank@cs.elte.hu

NOTATION

$\varrho_D(X)$: in-degree of a subset $X \subseteq V$ in a digraph $D = (V, A)$
$\delta_D(X)$: out-degree of a subset $X \subseteq V$ in a digraph $D = (V, A)$
$d_G(X)$: degree of a subset $X \subseteq V$ in an unditected graph $G = (V, E)$
$d_G(A, B)$: the number of edges connecting $A - B$ and $B - A$
$\Gamma(X)$: set of nodes outside $X$ that have at least one neighbour in $X$

NOTIONS

Tree, forests, arborescence, branching
Euler graph: $d(X)$ is even for every $X \subseteq V$
Euler digraph: $\delta(X) = \varrho(X)$ for every $X \subseteq V$
Smooth digraph: $|\varrho(v) - \delta(v)|$ for every node $v \in V$
Partially ordered set (poset), chain, antichain
strongly connected digraph
$k$-edge-connected graph or digraph
root-connected digraph: no dicut exists that is oriented toward a root $r_0$. Equivalently, every node is reachable from $r_0$

$k$-node-connected graph or digraph
one-way path or circuit in a digraph (sometimes: directed directed path/circuit or dipath/di-circuit)
$st$-path: a one-way path from $s$ to $t$
one-way cut (sometimes: directed cut or dicut) in a digraph
complete graph, complete bipartite graph, tournament
chromatic number, chromatic index
flows, ciculations
Totally unimodular (TU) matrix
Polyhedron, Polytope

THEOREMS

Gallai: in an edge-weighted digraph there is no one-way (directed) circuit of negative total weight if and only of there is a feasible potential
Kőnig-Hall on max matching
Menger on disjont paths
Max-flow Min-cut
Hoffman on feasible circulations
Kőnig's edge-colouring
Tutte on perfect matching
Tutte on maximum number of disjoint spanning trees
Nash-Williams on minimum number of covering forests
Edmonds on maximum number of disjoint arborescences
Dilworth on maximum antichain and minimum chain-decomposition of a poset
Mirsky (=polar Dilworth) on maximum chain and minimum antichain-decomposition
Farkas Lemma
Duality theorem of linear programming.

ALGORITHMS:

Depth-first search (DFS), Breadth-first search (BFS)
Greedy algorithms for min-cost tree.
Dijkstra
Bellman-Ford (or variant) for finding negative circuit or feasible potential.
Alternating path for maximum matchings in bipartite graph,
Max-flow Min-cut algorithm of Ford and Fulkerson