Research Institute for Discrete Mathematics
Chip Design
Summer term 2013

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Exercise Set 3

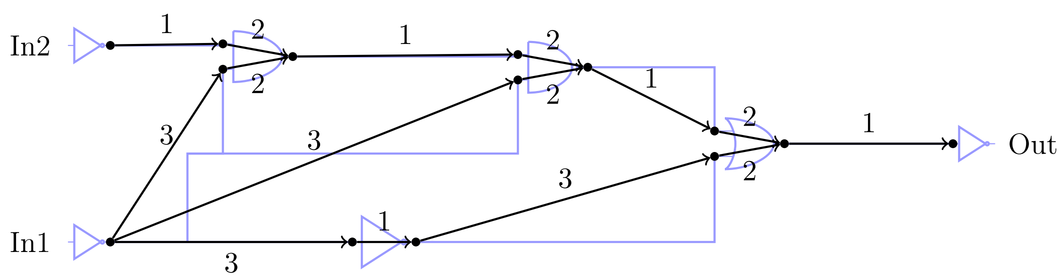
Exercise 3.1:
Let N be a finite set of pins, and let $\mathcal{S}(p)$ be a set of axis-parallel rectangles for each $p \in N$. We want to compute the bounding box net length of N. To this end, we look for an axis-parallel rectangle R with minimum perimeter such that for every $p \in N$ there is a $S \in \mathcal{S}(p)$ with $R \cap S \neq \emptyset$. Let $n := \sum_{p \in N} |\mathcal{S}(p)|$.
Show that such a rectangle can be computed in $\mathcal{O}(n^3)$ time.

Hint: Enumerate possible coordinates for the lower left corner of R.

(4 points)

Exercise 3.2:

Consider the following piece of combinational logic and its netlist graph:



The edge labels specify the delays. We do not distinguish between rising and falling signals and do not consider slew. Maximum (late mode) and minimum (early mode) delays are equal. Assume that all the arrival times for the latest and earliest signal at the primary inputs 'In1' and 'In2' are 0 and the required arrival times at the primary output 'Out' are 10 (early mode) and 12 (late mode).

a) What are the earliest and latest arrival times of a signal at the primary output pin 'Out' ?

b) Compute the early and late slack at each pin.

All definitions you need to solve the exercise can be found in the box on the next page.

(3 points)

Exercise 3.3:
Let $G = (V, E)$ be a netlist graph. Let $S \subset V(G)$ be the set of logical sources and let $T \subset V(G)$ be the set of logical sinks. For each source s, a signal σ_s starts in s at time $\mathrm{at}(\sigma_s)$. To traverse an edge $e = (p, q) \in E(G)$, a signals needs time $\mathrm{delay}^{\mathrm{late}}(e)$. For each $t \in T$, all signals have to arrive at time $\mathrm{rat}^{\mathrm{late}}(t)$.
As before, we do not distinguish between rising and falling signals and do not consider slews. We only consider late mode. Prove that

$$\begin{aligned}
&\mathrm{slack}^{\mathrm{late}}(p) \geq \min\{\mathrm{slack}^{\mathrm{late}}(q) \mid (p,q) \in \delta^+(p)\} \quad \text{and} \\
&\mathrm{slack}^{\mathrm{late}}(q) \geq \min\{\mathrm{slack}^{\mathrm{late}}(p) \mid (p,q) \in \delta^-(q)\}
\end{aligned}$$

(3 points)

Exercise 3.4:
For a finite nonempty set $T \subset \mathbb{R}^2$ we define

$$\begin{aligned}
\mathrm{BB}(T) &:= \max_{(x,y)\in T} x - \min_{(x,y)\in T} x + \max_{(x,y)\in T} y - \min_{(x,y)\in T} y \\
\textsc{Steiner}(T) &:= \text{length of a shortest rectilinear Steiner tree for } T \\
\mathrm{MST}(T) &:= \text{length of a minimum spanning tree in the complete graph on } T, \\
& \text{ where edge weights are } l_1\text{-distances}
\end{aligned}$$

Prove that

(a) $\mathrm{BB}(T) \leq \textsc{Steiner}(T) \leq \mathrm{MST}(T)$ for all finite sets $T \subset \mathbb{R}^2$.

(b) $\textsc{Steiner}(T) = \mathrm{BB}(T)$ for all $T \subset \mathbb{R}^2$ with $|T| \leq 3$

(c) $\textsc{Steiner}(T) \leq \frac{3}{2} \cdot \mathrm{BB}(T)$ for all $T \subset \mathbb{R}^2$ with $|T| \leq 5$

(d) there exists no $k \in \mathbb{R}$ with $\textsc{Steiner}(T) \leq k \cdot \mathrm{BB}(T)$ for all finite sets $T \subset \mathbb{R}^2$

(1+1+2+2 points)

Definition:
Let $G = (V, E)$ be a netlist graph, $p \in V(G)$ and let $\Sigma(p)$ be the set of signals at pin p. Each signal $\sigma \in \Sigma$ is associated with an arrival time $\mathrm{at}(\sigma) \in \mathbb{R}$. For each logical sink p we are given required arrival times $\mathrm{rat}^{\mathrm{early}}(p), \mathrm{rat}^{\mathrm{late}}(p) \in \mathbb{R}$. Timing rules require that $\mathrm{rat}^{\mathrm{early}}(p) \leq \mathrm{at}(\sigma) \leq \mathrm{rat}^{\mathrm{late}}(p)$ for each $\sigma \in \Sigma(p)$. Required arrival times can be computed backward by the recursion formulas

$$\begin{aligned}
\mathrm{rat}^{\mathrm{early}}(p) &= \max\{\mathrm{rat}^{\mathrm{early}}(q) - \mathrm{delay}^{\mathrm{early}}(e) \mid e = (p,q) \in E(G)\} \\
\mathrm{rat}^{\mathrm{late}}(p) &= \min\{\mathrm{rat}^{\mathrm{late}}(q) - \mathrm{delay}^{\mathrm{late}}(e) \mid e = (p,q) \in E(G)\}.
\end{aligned}$$

Furthermore, we define early and late mode slacks at a pin p as

$$\mathrm{slack}^{\mathrm{early}}(p) = \inf\{\mathrm{at}(\sigma) - \mathrm{rat}^{\mathrm{early}}(p) \mid \sigma \in \Sigma(p)\}, \quad \mathrm{slack}^{\mathrm{late}}(p) = \inf\{\mathrm{rat}^{\mathrm{late}}(p) - \mathrm{at}(\sigma) \mid \sigma \in \Sigma(p)\}.$$

Deadline: Thursday, May 2nd, before the lecture.