Research Institute for Discrete Mathematics
Chip Design
Summer Term 2014

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Exercise Set 5

Exercise 5.1:
Given a connected undirected graph $G = (V, E)$, a set $T \subseteq V$ with $|T| = 3$ and a cost function $c : E \to \mathbb{R}_{\geq 0}$, show how to compute a shortest Steiner tree for T in G in $\mathcal{O}(|V| \log |V| + |E|)$ time.

(4 points)

Exercise 5.2:
Let $T \subseteq \mathbb{R}^2$ be a finite set of terminals located on k parallel horizontal lines (i.e. $|\{y(t) : t \in T\}| = k$). We assume that the elements of T are sorted by their x-coordinate in non-decreasing order. Prove:

(a) If $k = 2$, a shortest rectilinear Steiner tree for T can be found in linear time.

(b) If k is constant and on each of the k parallel lines there is a terminal with x-coordinate $\min\{x(t) : t \in T\}$, a shortest rectilinear Steiner tree for T can be found in linear time.

(4 + 2 points)

Exercise 5.3:
Let Y be a Steiner tree for terminal set T in which all leaves are terminals. Prove that $\sum_{t \in T} \Big(|\delta_Y(t)| - 1\Big) = k - 1$, where k is the number of full components of Y.

(2 points)

Exercise 5.4:
Given a finite set $T \subseteq \mathbb{R}^2$, show how

a) $\textsc{Clique}(T) := \dfrac{1}{|T| - 1} \displaystyle\sum_{(x,y),(x',y') \in T} \Big(|x - x'| + |y - y'|\Big)$

b) $\textsc{Star}(T) := \displaystyle\min_{(x',y') \in \mathbb{R}^2} \sum_{(x,y) \in T} \Big(|x - x'| + |y - y'|\Big)$

can be computed in $\mathcal{O}(|T| \log |T|)$ time.

(2+2 points)

Deadline: Thursday, May 15, before the lecture.
The websites for lecture and exercises are linked at

http://www.or.uni-bonn.de/lectures/ss14/ss14.html

In case of any questions feel free to contact me at scheifele@or.uni-bonn.de .