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Exercise Set 3

Exercise 3.1. Assume unit B_2-circuit-delay and zero wire-delay.

(a) Show that for n inputs with arrival times $t_i \in \mathbb{N}$ $(i = 1, \ldots, n)$ there are n-ary AND, OR or XOR circuits over B_2 with delay $d \in \mathbb{N}$ if and only if

$$\sum_{i=1}^{n} 2^{t_i - d} \leq 1.$$

(b) Provide an algorithm that finds such a circuit in $\mathcal{O}(n \log n)$ time.

(3 + 3 points)

Exercise 3.2. Consider a prefix tree computing $z_n \circ \cdots \circ z_1$ for generate/propatage pairs $z_1, \ldots, z_n$ with arrival times $t_1, \ldots, t_n \in \mathbb{N}$, where $\circ$ is the prefix operator for adder circuits. Let F_k be the first Fibonacci number that is at least as large as $\sum_{i=1}^{n}(F_{t_i+3} - 1)$.

(a) Show that a prefix tree with B_2 delay at most k can be computed by computing a prefix tree for an instance with modified arrival times $t'_1, \ldots, t'_n \in \mathbb{N}$ with $\max\{t'_i : 1 \leq i \leq n\} \leq 2n - 1$.

(b) Assume linear-time addition and multiplication with constants. Show that for any fixed $\gamma > 1$ a prefix carry bit circuit with B_2-delay at most

$$\log_\varphi \left(\sum_{i=1}^{n} \varphi^{t_i} \right) + 4 + 2.1 \cdot n^{1-\gamma}$$

can be found in $\mathcal{O}(n\gamma \log^2 n)$ time where φ is the golden ratio.

(2 + 5 points)

Exercise 3.3. Let $n = 2^k$ for $k \in \mathbb{N}$ and a, b two n-bit numbers representing $|a|, |b| \in \mathbb{N}$. Define $f^n \in B_{2n,2n}$ as $f^n(a, b) := |a| \cdot |b|$ i.e. the product of two naturals.

(a) A *bit-shift* is a multiplication by 2^i for $i \in \mathbb{N}$. Show that $|a| \cdot |b|$ can be expressed in terms of at most 3 non-bit-shift multiplications of $\frac{n}{2}$-bit numbers, 6 additions of $2n$-bit numbers, and several bit-shifts.

(b) Show $S(n) \leq 3 \cdot S(\frac{n}{2}) + cn$ and $D(n) \leq D(\frac{n}{2}) + d \cdot \log_2 n$ for constants c and d.

(c) Let $\Omega := \{\wedge, \vee, \oplus\}$. Show $S_\Omega(f^n) = \mathcal{O}(n^{\log_2 3})$ and $D_\Omega(f^n) = \mathcal{O}(\log_2^2 n)$ for circuits with fanout 2.

(1 + 3 + 3 points)

Deadline: May 11th, before the lecture. The websites for lecture and exercises can be found at

`http://www.or.uni-bonn.de/lectures/ss17/chipss17.html`

In case of any questions feel free to contact me at ochsendorf@or.uni-bonn.de.