Exercise Set 5

Exercise 5.1.

(a) Prove that for any fixed $\varepsilon > 0$ there exists a polynomial-time algorithm which for any instance $I = (a_1, \dots, a_n)$ of the BIN PACKING problem finds a packing using the optimum number of bins but possibly violating the capacity constraints by ε, i. e. an $f : \{1, \dots, n\} \to \{1, \dots, \mathrm{OPT}(I)\}$ with $\sum_{f(i)=j} a_i \leq 1 + \varepsilon$ for all $j \in \{1, \dots, \mathrm{OPT}(I)\}$.

Hint: Use Exercercise 4.3.

(b) Use (a) to show that the MULTIPROCESSOR SCHEDULING PROBLEM (see Exercise 4.2) has an approximation scheme.

(8 points)

Exercise 5.2. Let $G = (V, E)$ be a graph with non-negative edge costs, and let $S \subseteq V$ and $R \subseteq V$ be disjoint vertex sets ("senders" and "receivers"). Consider the problem of finding a minimum cost subgraph of G that contains a path connecting each receiver to a sender.

(a) Prove that the restriction of this problem to instances with $S \cup R = V$ is in P.

(b) Prove that this problem is NP-hard and give a 2-factor approximation algorithm.

(4 points)

Exercise 5.3. Consider the DIRECTED STEINER TREE PROBLEM: Given a edge-weighted digraph $G = (V, E)$, a set of terminals $T \subseteq V$ and a root vertex $r \in V$, find a minimum weight arborescence rooted at r that contains every vertex in T.

Show that a k-approximation algorithm for the DIRECTED STEINER TREE PROBLEM can be used to obtain a k-approximation algorithm for SET COVER.

(4 points)

Professor Dr. Jens Vygen
Vera Traub

Exercise 5.4. Consider the restriction $\mathcal{P}$ of the unweighted Vertex Cover Problem to graphs where the maximum degree of every vertex is bounded by a constant B.

Let $\varepsilon > 0$. Show: If there exists a polynomial time approximation algorithm for the Steiner Tree Problem with performance ratio $1 + \epsilon$, then there exists a polynomial time approximation algorithm for problem $\mathcal{P}$ with performance ratio $1 + (B + 1)\epsilon$.

(4 points)

Deadline: Tuesday, May 29th, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss18/appr_ss18_ex.html`

In case of any questions feel free to contact me at traub@or.uni-bonn.de.