

Exercise Set 6

Exercise 6.1. Consider the STANDARD PLACEMENT PROBLEM on instances without blockages, where $h(C) \equiv 1 \equiv w(C)$ (unit size for $C \in \mathcal{C}$) as well as $w(N) \equiv 1$ (unit net weights for $N \in \mathcal{N}$).

Prove or disprove that this problem is NP-hard.

(5 points)

Exercise 6.2. Consider the *spreading LP* for $d = 2$:

$$\begin{aligned}
 \min \quad & \sum_{e \in E(G)} w(e) l(e) \\
 \text{s.t.} \quad & \sum_{y \in X} l(\{x, y\}) \geq \frac{1}{4} (|X| - 1)^{3/2} && x \in X \subseteq V(G) \\
 & l(\{x, y\}) + l(\{y, z\}) \geq l(\{x, z\}) && x, y, z \in V(G) \\
 & l(\{x, y\}) \geq 1 && x, y \in V(G), x \neq y \\
 & l(\{x, x\}) = 0 && x \in V(G)
 \end{aligned}$$

Show that the optimum of the spreading LP is a lower bound for the cost of any 2-dimensional arrangement.

(5 points)

Exercise 6.3. Let T be a finite, nonempty subset of \mathbb{R}^2 . Show that CLIQUE can be computed in $O(|T| \log |T|)$ time where

$$\text{CLIQUE}(T) := \frac{1}{|T| - 1} \sum_{\{(x,y), (x',y')\} \subseteq T} (|x - x'| + |y - y'|).$$

(4 points)

Exercise 6.4. Consider the following wirelength model for a $(\mathcal{C}, P, \gamma, \mathcal{N})$. For a net $N \in \mathcal{N}$,

$$\begin{aligned}
 \text{SmoothBB}(N) := & \ln \left(\sum_{p \in N} \exp(x(\gamma(p)) + x(p)) \right) + \ln \left(\sum_{p \in N} \exp(-x(\gamma(p)) - x(p)) \right) \\
 & + \ln \left(\sum_{p \in N} \exp(y(\gamma(p)) + y(p)) \right) + \ln \left(\sum_{p \in N} \exp(-y(\gamma(p)) - y(p)) \right).
 \end{aligned}$$

Prove:

(a) $BB(N) \leq \text{SmoothBB}(N) \leq BB(N) + 4 \ln |N|$.

(b) $\text{SmoothBB}(N)$ is a convex function in $(x_C)_{C \in \mathcal{C}}$.

(Hint: it is worth simplifying the notation before proving the core mathematical property.)

(2+3 points)

Deadline: May 19, before the lecture. The websites for lecture and exercises can be found at:

http://www.or.uni-bonn.de/lectures/ss22/chipss22_ex.html

In case of any questions feel free to contact me at blankenburg@or.uni-bonn.de.