Combinatorial Optimization
Winter term 2010/2011

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Exercises 4

Exercise 1:
Show that any simple graph with n vertices with minimum degree k has a matching of cardinality $\min\{k, \lfloor \frac{n}{2} \rfloor\}$.

(4 points)

Exercise 2:
Prove that an undirected graph G is factor-critical if and only if G is connected and $\nu(G) = \nu(G - v)$ for all $v \in V(G)$.

(4 points)

Exercise 3:
Recall the generic algorithm from set 3, excercise 2.

(a) Prove that – given a matching M – the union of all shortest M-augmenting paths in G can be found in $O(n + m)$ time.
Hint: Use a kind of breadth-first search with matching edges and non-matching edges alternating.

(b) Consider a sequence of iterations of the algorithm where the length of the augmenting path remains constant. Show that the time needed for the whole sequence is no more than $O(n + m)$.
Hint: First apply (a) and then find the paths successively by DFS. Mark vertices already visited.

(c) Combine (b) with Exercise 3.2(e) to obtain an $O(\sqrt{n}(m + n))$-algorithm for the Cardinality Matching Problem in bipartite graphs.

(6 points)

Deadline: Tuesday, November 9th, before the lecture.