Combinatorial Optimization
Winter term 2010/2011

Prof. Dr. Stefan Hougardy
Markus Struzyna

Exercises 9

Exercise 1:

Describe a polynomial-time algorithm for the following problem:
Given a graph G with weights $c : E(G) \to \mathbb{R}$ and $S, T \subseteq V(G)$, find a minimum weight set $F \subseteq E(G)$ such that $|\delta(v) \cap F|$ is even for $v \in S$ and odd for $v \in T$, or decide that no such set exists.

(4 points)

Exercise 2:

Let G be an undirected graph and $T \subseteq V(G)$ with $|T|$ even. Prove that the convex hull of the incidence vectors of all T-joins in G is the set of all vectors $x \in [0,1]^{E(G)}$ satisfying

$$\sum_{e \in \delta_G(X) \setminus F} x_e + \sum_{e \in F} (1 - x_e) \geq 1$$

for all $X \subseteq V(G)$ and $F \subseteq \delta_G(X)$ with $|X \cap T| + |F|$ odd.

(4 points)

Exercise 3:
Let G be an undirected graph, $T \subseteq V(G)$ with $|T|$ even, and $F \subseteq E(G)$.
Prove:

(a) F has nonempty intersection with every T-join if and only if F contains a T-cut.

(b) F has nonempty intersection with every T-cut if and only if F contains a T-join.

(4 points)

Deadline: Tuesday, December 14th, before the lecture.