Winter term 2013/14 Research Institute for Discrete Mathematics
Prof. Dr. Stefan Hougardy University of Bonn
Niko Klewinghaus

Combinatorial Optimization

Exercise Sheet 8

Exercise 8.1:
Describe a polynomial time algorithm for the following problem: Given an undirected graph G with weights $c : E(G) \to \mathbb{R}$ and $S, T \subseteq V(G)$, find a minimum weight set $F \subseteq E(G)$ such that $|\delta(v) \cap F|$ is even for all $v \in S$ and odd for all $v \in T$ or decide that no such set exists.

(4 points)

Exercise 8.2:
Let G be a bipartite graph and $J \subseteq E(G)$. Prove: J satisfies $|J \cap E(C)| \leq \frac{1}{2}|E(C)|$ for each circuit C if and only if there are $|J|$ disjoint cuts each intersecting J in exactly one edge.

(4 points)

Exercise 8.3:
Let G be a simple graph with $|V(G)| \geq 2$ and $|\delta(v)| \geq k$ for all $v \in V(G)$. Prove that there are two vertices s and t such that there exist at least k edge-disjoint s-t-paths in G. Is this still true if there is exactly one vertex v with $|\delta(v)| < k$?

(4 points)

Exercise 8.4:
Let G be an undirected graph and $T \subseteq V(G)$ with $|T|$ even. Prove that the convex hull of the incidence vectors of all T-joins in G is the set of vectors $x \in [0,1]^{E(G)}$ satisfying

$$\sum_{e \in \delta_G(X) \setminus F} x_e + \sum_{e \in F} (1 - x_e) \geq 1$$

for all $X \subseteq V(G)$ and $F \subseteq \delta_G(X)$ with $|X \cap T| + |F|$ odd.
Hint: Use Theorem 56 and Theorem 50.

(4 points)

Deadline: Thursday, December 12, 2013, before the lecture.