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# Exercise Set 4

**Exercise 4.1.** Consider the Shortest Even/Odd Path Problem: Given a graph $G$ with weights $c : E(G) \to \mathbb{R}_{\geq 0}$ and $s, t \in V(G)$, find an $s$-$t$-path $P$ of even/odd length in $G$ that minimizes $\sum_{e \in E(P)} c(e)$ among all $s$-$t$-paths of even/odd length in $G$. Show that both the even and the odd version can be linearly reduced to the Minimum Weight Perfect Matching Problem.

(5 points)

**Exercise 4.2.** Consider the Minimum Cost Edge Cover Problem: Given a graph $G$ with weights $c : E(G) \to \mathbb{R}_{\geq 0}$, find an edge cover $F \subseteq E(G)$ that minimizes $\sum_{e \in F} c(e)$. Show that the Minimum Cost Edge Cover Problem can be linearly reduced to the Minimum Weight Perfect Matching Problem.

(5 points)

**Exercise 4.3.** Let $G$ be a graph with edge weights $c : E(G) \to \mathbb{R}$ and let $M$ be a matching in $G$ with $|M| = k$ that has minimum weight among all matchings in $G$ that contain exactly $k$ edges. Let $P$ be an $M$-augmenting path in $G$ with minimum gain. Let $M' := M \triangle E(P)$. Prove that $M'$ has minimum weight among all matchings in $G$ that contain exactly $k+1$ edges.

(5 points)

**Exercise 4.4.** Let $G$ be an undirected graph with edge weights $c \colon E(G) \to \mathbb{R}$, and let $M$ be a matching so that $c(N) \leq c(M)$ for all matchings $N$ in $G$ with $|M| - 1 \leq |N| \leq |M| + 1$. Prove that then $M$ is a maximum weight matching in $G$.

(5 points)

**Deadline:** November 7$^{\text{th}}$, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ws19/co_exercises/exercises.html`

In case of any questions feel free to contact me at rabenstein@or.uni-bonn.de.