Exercise Set 10

Exercise 10.1. Let $n \in \mathbb{N}$. For which values of k is the uniform matroid of rank k on the set of n elements the graphic matroid of some simple graph?

(3 points)

Exercise 10.2. Let E be a finite set and $P \subseteq \mathbb{R}^E$ be a polymatroid. Show that there is some submodular set function f with $f(\emptyset) = 0$, f monotone, i.e. $f(X) \leq f(Y)$ for all $X \subseteq Y \subseteq E$ and $P = P(f)$.

(5 points)

Exercise 10.3. Let E be a finite set, $f\colon 2^E \to \mathbb{R}$ a submodular set function and f' its Lovasz extension. For all $x \in [0,1]^E$, show

$$f'(x) = \max\{x^T y \colon y \in P(f)\}$$

(6 points)

Exercise 10.4. Let E be a finite set, $f\colon 2^E \to \mathbb{R}$ a set function and f' its Lovasz extension. Prove that f is submodular if and only if f' is convex.

(6 points)

Deadline: December 19$^{\text{th}}$, before the lecture. The websites for lecture and exercises can be found at:

http://www.or.uni-bonn.de/lectures/ws19/co_exercises/exercises.html

In case of any questions feel free to contact me at rabenstein@or.uni-bonn.de.