Winter term 2021/22
Research Institute for Discrete Mathematics
Prof. Dr. B. Korte
University of Bonn
Dr. U. Brenner

Combinatorics, Graphs, Matroids
Assignment Sheet 1

1. Let $\mathcal{S}$ be a finite family of finite (not necessarily pairwise disjoint) sets. A set T is called *transversal* of $\mathcal{S}$ if there is a bijection $\Phi : T \to \mathcal{S}$ with $t \in \Phi(t)$ for all $t \in T$. Assume that $\mathcal{S}$ contains at least one transversal. Prove that the set of all transversals of $\mathcal{S}$ is the family of bases of a matroid (the so-called *transversal matroid*). (4 points)

2. Let G be a graph, and let $\mathcal{F}$ be the family of all sets $X \subseteq V(G)$ such that a matching with maximum cardinality exists that does not cover any node in X. Show that $(V(G), \mathcal{F})$ is a matroid. (4 points)

3. Let $(E, \mathcal{F}_1)$ and $(E, \mathcal{F}_2)$ be two matroids and $k \in \mathbb{N}$. Which of the following set systems are necessarily matroids? Prove the correctness of your answers.
   (a) $(E, \mathcal{F}_1 \cup \mathcal{F}_2)$
   (b) $(E, \mathcal{F}_1 \cap \mathcal{F}_2)$
   (c) $(E, \mathcal{F}_1 \cap \{X \subseteq E \mid |X| \leq k\})$ (2+2+2 points)

4. Let $(E, \mathcal{F})$ be a matroid with rank function r. Prove or disprove the following statement:
   $(E, \mathcal{F})$ is uniform (i.e. there is a k such that $\mathcal{F} = \{F \subseteq E \mid |F| \leq k\}$) if and only if there is no circuit with less than $r(E) + 1$ elements. (2 points)

Du date: Thursday, October 21, 2021, before the lecture (in the lecture hall)