Winter term 2021/22
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Combinatorics, Graphs, Matroids
Assignment Sheet 4

1. Prove or disprove the following statement: A matroid is a transversal matroid (see exercise 1 of assignment sheet 1) if and only if it is the union of matroids whose bases each have cardinality 1. (2 points)

2. Let $(E, \mathcal{F}_1)$ and $(E, \mathcal{F}_2)$ be two matroids. And let X be a maximum partitionable set with respect to $(E, \mathcal{F}_1)$ and $(E, \mathcal{F}_2^*)$, so in particular we have $X = X_1 \dot{\cup} X_2$ with $X_1 \in \mathcal{F}_1$ and $X_2 \in \mathcal{F}_2^*$. Moreover, let $B_2^* \supseteq X_2$ be a basis of $\mathcal{F}_2^*$. Show that $X \setminus B_2^*$ is a maximum-cardinality set in $\mathcal{F}_1 \cap \mathcal{F}_2$. (5 points)

3. Let $(E, \mathcal{F})$ be a matroid with rank function r. Show the following statements:
   (a) $(E, \mathcal{F})$ has k pairwise disjoint bases if and only if $kr(A) + |E \setminus A| \geq kr(E)$ for all $A \subseteq E$.
   (b) In $(E, \mathcal{F})$, there are k independent sets whose union is E if and only if $kr(A) \geq |A|$ for all $A \subseteq E$. (3+3 points)

4. Consider the following problem: Given a simple undirected connected graph G with edge weights $c : E(G) \to \mathbb{N}$, find an edge set $F \subseteq E(G)$ with maximum weight such that $(V(G), E(G) \setminus F)$ is connected and $(V(G), F)$ does not contain a circuit. Show that the is an algorithm for this problem whose running time is polynomial in the input size. (3 points)

Du date: Thursday, November 11, 2021, before the lecture (in the lecture hall)