Winter term 2021/22
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Combinatorics, Graphs, Matroids
Assignment Sheet 6

1. Let the graph G be planar but assume that adding any edge between two nodes that are not adjacent in G leads to a non-planar graph. Is it true that for each two nodes u,v that are not adjacent in G the graph $(V(G), E(G) \cup \{\{u, v\}\})$ contains K_5 as a minor? Prove the correctness of your answer. (4 points)

2. Let G be a connected graph with planar embedding Φ. Let G^* be its planar dual. Show that the number of spanning trees is the same for G and G^*. (3 points)

3. *triangulation* is a planar graph together with a planar embedding in which each region is bounded by a triangle. For the node set V of a triangulation let $l : V \to \{1, 2, 3\}$ be a mapping. A face of the triangulation is called tri-colored if at its three corner nodes all three different node labels 1, 2, and 3 occur. Show that there must be an even number of tri-colored faces. (3 points)

4. Consider the greedy node coloring algorithm, in which the nodes are traversed in some order and each node is given the smallest color not yet used on its already colored neighbors. Show that for any $n \geq 2$ there is a graph G with $|V(G)| = 2n$ and $\chi(G) = 2$ such that if the nodes are traversed in an appropriate order, the greedy algorithm needs n colors. Conversely, show that for each graph G there is an order of the nodes such that if the greedy algorithm considers the nodes in that order, it needs only $\chi(G)$ colors. (2 points)

5. Let G be an undirected non-complete graph. Show that there is a partition $V(G) = V_1 \dot{\cup} V_2$ such that $\chi(G[V_1]) + \chi(G[V_2]) > \chi(G)$. (4 points)

Du date: Thursday, November 25, 2021, before the lecture (in the lecture hall)