Winter term 2021/22
Prof. Dr. B. Korte
Dr. U. Brenner

Research Institute for Discrete Mathematics
University of Bonn

Combinatorics, Graphs, Matroids
Assignment Sheet 8

1. Consider a regular 250-gon, of which exactly 16 corners are colored red while all others are colored blue. Show that there is a rotation of the 250-gon such that all red corners are rotated to old positions of blue corners. (3 points)

2. In a tennis tournament, two teams with n players each play against each other in the following way: The players of both teams are numbered from 1 to n. First, the two players with the number 1 play against each other. Whoever loses is then always eliminated and is replaced in the following round by the player of his team with the next highest number. In the second round, the winner of the first round plays against player 2 of the opposing team. The team that runs out of players first, i.e. the team that lost n games first, loses the tournament. How many possible tournament outcomes, i.e. sequences of wins and losses, are there in this system? (3 points)

3. Let $B_0 = 1$ and $B_n = \sum_{k=0}^{n} S_{n,k}$ for $n \in \mathbb{N} \setminus \{0\}$. Show:

$$B_{n+1} = \sum_{k=0}^{n} \binom{n}{k} B_k.$$

(3 points)

4. Prove:

   (a) $s_{n+1,k+1} = \sum\limits_{i=k}^{n} \binom{i}{k} s_{n,i}$,

   (b) $S_{n+1,k+1} = \sum\limits_{i=0}^{n} \binom{n}{i} S_{i,k}$. (2+2 points)

5. For $m, n \in \mathbb{N}$ we define $X_n := \{1, \ldots, n\}$ and

$$A_{n,m} := \left| \left\{ \pi : X_n \to X_n : \pi \text{ permutation and } |\{i \in X_n \setminus \{n\} : \pi(i) < \pi(i+1)\}| = m \right\} \right|.$$

Moreover, let $A_{0,0} := 1$ and $A_{0,k} := 0$ (for $k > 0$). Show how $A_{n,m}$ for $n > 0$ and $m > 0$ can be computed from $A_{n-1,m-1}$ and $A_{n-1,m}$ by a constant number of arithmetic operations. (3 points)

Du date: Thursday, **December 16**, 2021, before the lecture (in the lecture hall)