

## Combinatorics, Graphs, Matroids

### Assignment Sheet 9

1. Let  $S_{n,k}^o$  be the number of ways to decompose the set  $\{1, \dots, n\}$  into  $k$  subsets such that each of these subsets has odd size. Find a recursive formula that allows to calculate  $S_{n+1,k+1}^o$  from the values  $S_{n',k'}^o$  with  $n' \leq n$  and  $k' \leq k$ . (2 points)
2. Let  $n$  be a positive integer. Show that there is a positive integer  $N$  which is an integer multiple of  $n$  and whose decimal representation contains only the digits 3 and 0. (3 points)
3. At a party, nine people meet, none of whom is younger than one year and none of whom is older than 60. Show that you can then find two (disjoint) groups of guests such that the sums of the ages in both groups are equal. (2 points)
4. Prove the general version of Ramsey's theorem: Given  $k$  and  $l_1, \dots, l_r$ , there is a smallest number  $R(k; l_1, \dots, l_r)$  such that the following statement holds: If  $N$  is a set with  $n$  elements with  $n \geq R(k; l_1, \dots, l_r)$  and the  $k$ -element subsets of  $N$  are somehow colored with colors  $1, \dots, r$ , then there exists a color  $i$  such that in a  $l_i$ -element subset of  $N$  all  $k$ -element subsets are colored with  $i$ . (5 points)  
Hint: Apply induction on  $r$ . For  $r = 2$  an induction on  $k$  is suitable.

**Due date: Thursday, January 13, 2022, before the lecture (in the lecture hall)**

**Information from the Student Council:** This year the Maths Christmas Party will take place virtually on Tuesday, December 21, starting at 18 c.t.. Latest information and a registration can be found on our website ([fsmath.uni-bonn.de](https://fsmath.uni-bonn.de)). Come by!