

## Combinatorics, Graphs, Matroids

### Assignment Sheet 10

1. Calculate for  $x \neq 1$  the following expressions by the method “isolating the terms” (i.e. find a representation that allows an evaluation with a constant number of arithmetic operations):

(a)  $\sum_{k=1}^n kx^k$

(b)  $\sum_{k=1}^n k^2 x^k$  (2+2 points)

2. Let  $T_0 = 0$ ,  $T_1 = 1$  and  $T_n = -3T_{n-1} - 2T_{n-2}$  for  $n > 1$ . By choosing appropriate summation factors, give a formula to directly calculate  $T_n$ . (4 points)
3. Using partial summation, show how  $\sum_{k=1}^n H_k^2$  can be calculated from  $H_n$  and  $n$  with a constant number of arithmetic operations. (4 points)
4. Determine the connection coefficient of the bases  $\{x^{\bar{n}}\}$  and  $\{x^{\underline{n}}\}$ , i.e. find numbers  $a_{n,k}$  and  $b_{n,k}$  (for  $n, k \in \mathbb{N} \cup \{0\}$ ), such that for all  $n, k \in \mathbb{N} \cup \{0\}$  we have:

$$x^{\bar{n}} = \sum_{k=0}^n a_{n,k} \cdot x^{\underline{k}} \quad \text{and}$$

$$x^{\underline{n}} = \sum_{k=0}^n b_{n,k} \cdot x^{\bar{k}}$$

(4 points)

Hint: Use that for complex  $x$  and  $y$  and  $n \in \mathbb{N} \cup \{0\}$  the Vandermonde identity holds, i.e.

$$\binom{x+y}{n} = \sum_{k=0}^n \binom{x}{k} \binom{y}{n-k}.$$

**Due date: Thursday**, January 20, 2022, before the lecture (in the lecture hall)

#### Event notice of the gender equality committee:

On January 15, 2022, from 3.00 p.m. to 5.30 p.m. we invite all female<sup>1</sup> math and computer science students to an online event on the topic of “Networking and Career Paths”. Register here:

<http://www.hcm.uni-bonn.de/networking-career-paths/>

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<sup>1</sup>All female, intersexual, non-binary, transgender and agender persons are invited.