Combinatorics, Graphs, Matroids
Assignment Sheet 11

1. For $n \in \mathbb{N} \setminus \{0\}$ let $\Lambda(n)$ be the number of undirected graphs on the vertex set $\{1,\ldots,n\}$ where no vertex has degree 0. Give a formula for calculating $\Lambda(n)$.

2. Determine the generating function of the harmonic numbers.
   Hint: Use the following equation known from calculus: $\sum\limits_{n\geq 1} \frac{(-1)^{n+1}z^n}{n} = \log(1+z)$.

3. For constants $b, c, d \in \mathbb{R}$, let the sequence $(a_n)_{n\geq 0}$ be given by $a_n = ba_{n-1} + cd^{n-1}$ for $n \geq 1$ and $a_0 = 0$. Find a closed-form formula to compute the elements of the sequence.

4. Determine the number y_n of words of length n over the alphabet $\{1, 2, 3\}$ that contain an even number of ones and an odd number of twos.
   Hint: Use exercise 3.

The solutions to these exercises will not be marked any more.