

Combinatorics, Graphs, Matroids

Assignment Sheet 2

1. Let $\tilde{S}_{n,k}$ the number of ways to partition a set of n elements into k sets such that every set contains at least two elements.
 - (a) Determine $\tilde{S}_{2k,k}$.
 - (b) Find a recursion formula for $\tilde{S}_{n,k}$ with $n > 2k$. Prove the correctness of your formula.
(2+2 points)
2.
 - (a) A beaver cuts down at least one tree every day for a period of 90 days, but no more than 150 trees in total. Show that then for every $k \in \{1, \dots, 29\}$ there is a period of consecutive days when the beaver cuts down exactly k trees.
 - (b) Nine people meet at a party, none of whom is younger than one year and none of whom is older than 60. Show that you can find two (disjoint) groups of guests such that the sums of the ages in both groups are equal.
(2+2 points)
3. Determine the Ramsey number $R(3, 4)$.
(3 points)
4. Prove the general version of Ramsey's Theorem: Let k and $l_1, \dots, l_r$ be given. Then there exists a smallest number $R(k; l_1, \dots, l_r)$ such that the following holds: If N is an n -element set with $n \geq R(k; l_1, \dots, l_r)$ and the k -element subsets of N are colored with the colors $1, \dots, r$, then there is a color i such that there is an l_i -element subset of N , in which all k -element subsets are colored with i .
(5 points)
Hint: Use induction on r . For $r = 2$, induction on k can be useful.

You will always find the current exercise sheet on the eCampus page of the lecture.

Due date: Thursday, October 30, 2025, 12:00, via the eCampus exercise group.

Web pages:

https://ecampus.uni-bonn.de/goto_ecampus_crs_3882825.html

http://www.or.uni-bonn.de/lectures/ws25/kgm_ws25.html